

Comparison of the Geometrical Characters Inside Quark- and Gluon-jet Produced by Different Flavor Quarks

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Abstract

The characters of the angular distributions of quark jets and gluon jets with different flavors are carefully studied after introducing the cone angle of jets. The quark jets and gluon jets are identified from the 3-jet events which are produced by Monte Carlo simulation Jetset7.4 in e^+e^- collisions at $\sqrt{s}=91.2\text{GeV}$. It turns out that the ranges of angular distributions of gluon jets are obviously wider than that of quark jets at the same energies. The average cone angles of gluon jets are much larger than that of quark jets. As the multiplicity or the transverse momentum increases, the cone-angle distribution without momentum weight of both the quark jet and gluon jet all increases, *i.e* the positive linear correlation are present, but the cone-angle distribution with momentum weight decreases at first, then increases when $n > 4$ or $p_t > 2\text{GeV}$. The characters of cone angular distributions of gluon jets produced by quarks with different flavors are the same, while there are obvious differences for that of the quark jets with different flavors.

Keyword e^+e^- collisions; quark jets and gluon jets; Geometrical characters.

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1 Introduction

According to quantum chromodynamics(QCD), the basic elements that constitute substances are quarks and gluons. Because of "color confinement", we could not find free quarks and gluons. However, through analyzing the hadronization production from quarks and gluons, the characters of quarks and gluons can be obtain indirectly.

In 1975, 2-jet events were found for the first time in e^+e^- collisions[1]. After this, in 1979, 3-jet events were observed in the energy range $17 - 30\text{GeV}$ in e^+e^- collisions[2]. According to local parton-hadron dualism (LPHD)[3], hadron jets can reflect information of the decay and hadronization of original partons. Thereby through studying on jets, we can indirectly get characters of strong interactions among quarks and gluons.

The viewpoint that flavor quantum number is independent of strong interaction is the basic properties of QCD, and the only reason for destroying flavor symmetry is the "mass effect" in heavy quarks decaying. For the color charge of gluon is larger than that of quark,

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gluon jet has the character that it is fatter than quark jet. The experimental result [4]–[11] obtained in LEP e^+e^- storage ring (CERN) is quantificationally in agreement with the theoretically predictions: gluon jet has larger average charged particle multiplicity, softer fragmentation function and wider angular distributions than quark jet [12]. Recently, the general characteristics [13]–[16] of and the dynamical fluctuations inside [17]–[18] quark and gluon jets from three-jet events have been analyzed using Monte Carlo simulation in e^+e^- collisions.

In this thesis, we study the geometrical characters of quark jets and gluon jets and compare the differences between quark jets and gluon jets, starting from the characters of angular distributions; and analysed and compared carefully the geometrical characters of jets formed by different flavor quarks and them emitted gluons with the aim of studying the difference among these kinds of jets, which the flavor of the original quark is discriminated by changing parameter of model.

The event samples of final state particles of e^+e^- collisions at $\sqrt{s} = 91.2\text{GeV}$ are produced by Monte Carlo Simulation Jetset7.4 generator. The three-jets sub-samples are obtained by Durham jet-algorithm [21] from the full event samples [22]. According to QCD, the three-jets are separately produced by the hadronization of the original quark and anti-quark produced in e^+e^- collision and a hard gluon emitted by one of the original quark (or anti-quark). The original quark (or anti-quark) which had emitted this hard gluon is called mother quark. For convenience of comparing characters of quark jets and gluon jets with the same conditions, in this thesis we studied jets produced by mother quarks and gluons. Hereinafter the mother quark jet is just called quark jet for short, and jets formed by quark is called quark jet with flavor of this quark. For example, jet formed by b-quark is called b-quark jet. To check whether there is any character difference among gluons emitted by quarks with different flavors, we especially distinguished the gluons: a gluon emitted by one quark with certain flavor is called gluon with this kind of quark flavor, and correspondingly jet formed through this gluon hadronization is called gluon jet of this quark kind. Such as, one hard gluon emitted by a b-quark is called b-gluon; and jet formed though this b-gluon hadronization is called b-gluon jet.

To study the properties of quark jets and gluon jets, we also need to identify the two quark jets and one gluon jet from the three jets in a 3-jet event. We chose the angular method [23], which can be simply expressed with fig.1. Where, $P_i (i = 1, 2, 3)$ is the sum of

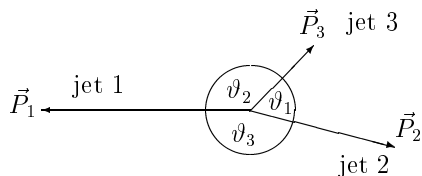


Figure 1: The skeleton sketch of 3-jet event distribution

momentum of all particles in jet- i . The angles between each two jets are defined as

$$\theta_i = \arccos \left(\frac{P_{j1}P_{k1} + P_{j2}P_{k2} + P_{j3}P_{k3}}{P_j P_k} \right), (i, j, k = 1, 2, 3; i \neq j, j \neq k, k \neq i). \quad (1)$$

Where jet facing the largest angle θ_3 is the gluon jet, jet facing the smallest angle θ_1 is the jet formed by the original quark which had not emitted hard gluon, and jet facing the middle angle θ_2 is the mother quark jet. Considering the requirement of momentum conservation, thus the three jets should lie in one plane, we added one condition: $\theta_1 + \theta_2 + \theta_3 \geq 359^\circ$. To improve the purity of selected events[24], we added condition: $\theta_3 - \theta_2 \geq 10^\circ$.

The thrust frame(as axis z) is commonly used for the three-dimensional phase space in e^+e^- collisions [25]. In order to get the characters of angular distributions inside quark jets and gluon jets more accurately, the jet frame is constructed taking the total momentum of jet as the longitudinal axis (axis z) [26].

2 The 2-dimensional angular distributions of particles inside jets

To describe the angular distribution characters of particles inside jets, we defined two angular distribution variables α_1 and α_2 ,

$$\begin{aligned} \alpha_1 &= f_1(\theta, \phi) = P_x/P = \sin \theta \cos \phi, \\ \alpha_2 &= f_2(\theta, \phi) = P_y/P = \sin \theta \sin \phi. \end{aligned} \quad (2)$$

Where θ is the angle between the particle momentum direction and the jet momentum direction, and ϕ is the angle between particle momentum projection in transverse plane and x -axis direction.

In our Monte Carlo simulation, a total number of 5000,000 events of e^+e^- collisions at 91.2 GeV are produced by Jetset 7.4 generator and the numbers of 379,825 3-jet events are selected out using Durham algorithm, then we separately selected out quark jets and gluon jets with energy at 24GeV, 28GeV from 3-jet events. According to the definition of angular distribution variables in equation (2), we plot the the 2-dimensional angular distributions of particles in quark jets and gluon jets with different jet energy, as it is shown in Fig.2. Fig.2 provides us column diagram of 2-dimensional angular distributions of quark (without distinguishing the original quark flavors) jets and gluon jets with energy at 24GeV and 28GeV, respectively.

It can be seen from Fig.2 that the angular distribution range of particles inside gluon jets are obviously wider than that inside quark jets with the same energy, and the distribution has perfect symmetry relative to jet axis, namely, the space distribution of jets present as taper. All these conclusions is consistent with the predictions of QCD theory.

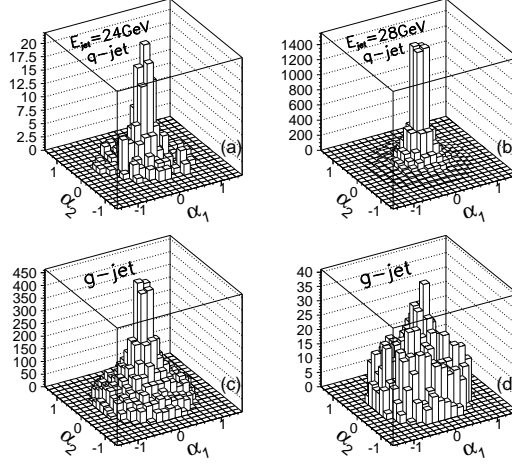


Figure 2: The 2-dimensional angular distribution column diagram of particles inside quark jets and gluon jets. The first line is the quark jet, and the second line is the gluon jet; the first column the energy is 24GeV, the second column the energy is 28GeV.

3 Definition of cone angle

In the previous section, we qualitatively discussed the angular distribution characters of particles inside jets and compared the angular distribution characters of quark jets and gluon jets. To quantificationally study the geometrical characters of jets, jet cone angle is defined as follows[27].

Suppose the number of charged particles of jet- i in one event is n_i , the total momentum of this jet is P^i , the momentum of the j -th charged particle in this jet is P_j^i , thus the cone angle of this jet is defined as

$$\langle \theta \rangle = \frac{1}{n_i} \sum_{j=1}^{n_i} \arccos\left(\frac{\vec{P}^i \cdot \vec{P}_j^i}{|\vec{P}^i| |\vec{P}_j^i|}\right). \quad (3)$$

Considering status of particles with momentum of different magnitudes is different in jets, namely, because they have different effect on the jet, we added momentum weight ω_j while calculating cone angles of jets:

$$\omega_j = \frac{|\vec{P}_j^i|}{\frac{1}{n_i} \sum_{k=1}^{n_i} |\vec{P}_k^i|}. \quad (4)$$

Thus cone angle with momentum weight of jet is defined as

$$\langle \theta \rangle = \frac{1}{n_i} \sum_{j=1}^{n_i} \omega_j \arccos\left(\frac{\vec{P}^i \cdot \vec{P}_j^i}{|\vec{P}^i| |\vec{P}_j^i|}\right). \quad (5)$$

4 The cone angular distributions of quark jets and gluon jets

In order to study the cone-angle property of jets, four event samples of final state particles with 5,000,000 events are generated from Jetset 7.4 generators for e^+e^- collisions at c.m. energy 91.2 GeV. Then the events of number 379825 for 3-jets are obtained by Durham algorithm from the full event samples, and the single quark jet and single gluon jet sub-samples are selected from 3-jet events using the angular rule with energy at 18GeV and 24GeV, respectively. Using these sub-samples we can analysis and compare various properties for quark- and gluon- jets.

The distributions of cone angle defined in Eq's. (3) and (5) are shown in Fig.3 for quark- and gluon- jets with energy at 18GeV and 24GeV, respectively. It can be seen from Fig.3 (a), with the energy of jet at 18GeV, that the cone angle of gluon-jet are distributed over $1^\circ - 50^\circ$ with an average value equal to 24.6° ; while the cone angle of quark-jet take values in the region $1^\circ - 45^\circ$ and the average is only 16.2° . When the energy of jet are 24GeV, showing in Fig.3 (b), the distribution of cone angle for gluon-jet and quark-jet move slightly to right to that of the cone-angle with energy at 18GeV.

The distribution of cone-angle of jet with the momentum weight is shown in Fig.3 (c) and (d). It can be seen through comparing Fig's.3 (a),(b) and (c),(d) that the distribution of weighted cone-angle is moved leftward for $4.8^\circ - 6.3^\circ$ in comparison to that of the unweighted cone-angle. The reason is because the momenta of particles nearby the jet axis is generally bigger than those far from the jet axis. The relation between the distributions of quark- and gluon- jets are qualitatively the same for weighted and unweighted cone-angles.

5 Comparison of correlation characters with average cone angles

We use Monte Carlo generator Jetset7.4 to produce 5,0000,000 e^+e^- collision events at an energy $\sqrt{s} = 91.2\text{GeV}$. The event samples are constructed according to the flavors of the original quark-pair $b\bar{b}, c\bar{c}, d\bar{d}, s\bar{s}, u\bar{u}$, respectively. The 3-jet event sub-samples are selected out using Durham rules with the cut parameter $y_{cut} = 0.002$, and the quark-jets(mother quark-jets) and gluon-jets are selected out from 3-jet events using angular method.

In order to study the correlation of cone angle of jets with other jet-variables, *e.g.* jet multiplicity — the number of charged particles in a jet. we divide the multiplicity region $n_l = 2 \sim 12$ into 11 bins and separately calculate the average cone angles of jets in each multiplicity bin.

$$\langle \theta_C \rangle_{l,k} = \frac{1}{n_l} \sum_{m=1}^{n_l} \langle \theta_C \rangle_{jet-k}^{l,m}, \quad (k = 1, 2, 3; l = 2, \dots, 12). \quad (6)$$

Where $\langle \theta_C \rangle_{jet_k}^{l,m}$ is the cone angle of the m th jet of kind k with charge multiplicity in the multiplicity bin l th. The n_l is the number of jet in the l th multiplicity bin, $\langle \theta_c \rangle_{l,k}$ is the cone-angle of the jet of kind k jet in the l th bin. The results of average cone-angle of the gluon jet and quark jet *vs.* multiplicity in each multiplicity bin for Jetset 7.4 Monte Carlo generators are shown in Fig's 4 (a) and (b), respectively.

It can be seen from the Fig 4 (a) that the average cone-angles of the gluon jet and quark jet *vs.* multiplicity act differently. The average cone-angle of the gluon jet is bigger than that of the quark jet, providing further evidence that the gluon jet is "fatter" than quark jet. The average cone-angle of the quark jet and gluon jet all increases with multiplicity.

The results of cone-angles with momentum weight and unweight are some quantificationally difference. The fig 4 (a) shows that the cone-angle of gluon jet and quark jet without momentum weight increases linear as the increasing of multiplicity, i.e the cone-angle of gluon jet and quark jet are positive correlation with multiplicity. The fig 4 (b) shows that the cone-angle of gluon jet and quark jet with momentum weight decreases as the increasing of multiplicity for $n < 4$, and after that the cone-angles of gluon jet and quark jet increase with the increasing of multiplicity when $n > 4$, developing a valley, i.e the cone-angle of gluon jet and quark jet are negative correlation with multiplicity as $n < 4$ and positive correlation as $n > 4$. This appearance existing minimum values after taking momentum weight into consideration is mainly aroused by the leading particles effect. When $n = 2$, the jet axis lies between the two particles and the average cone angle is relatively large; when $n = 3$ or $n = 4$, as momentum of the leading particles is larger than the other particles and it closes with the jet axis, the average cone angles of jet become smaller after considering momentum weight; when $n > 4$, for number of particles gradually become more, momentum of leading particles become less and effects to average cone angles become weaker, so the average cone angle increases with the increasing of charged particle multiplicity as $n > 4$.

It is remarkable that the average cone-angle of the gluon jet with different flavors are equation within range error, which illuminate that gluons jet produced by hard gluon emitted by quarks with different flavors have certain same characters; while that of the quark jet with different flavors are qualitatively conformable and are quantificationally difference, which illustrates that characters of different flavors quarks are different. This kind of difference is mainly aroused by mass difference among different flavors quarks.

Now we turn to the correlation between cone-angle and transverse momentum. We divide the transverse momentum range $P_t = 0 \sim 10$ GeV/c into 12 bins, and calculate the average cone-angle with different flavors quark jets and gluon jets in each transverse momentum bin. When calculating the average cone angles in each transverse momentum intervals, we separately made statistics of the total cone angle of one certain kind of jets in transverse momentum bin l , divided it by the number n_l of this certain kind of jets in this transverse momentum bin, then we got the average cone angle of this certain kind of jets in this

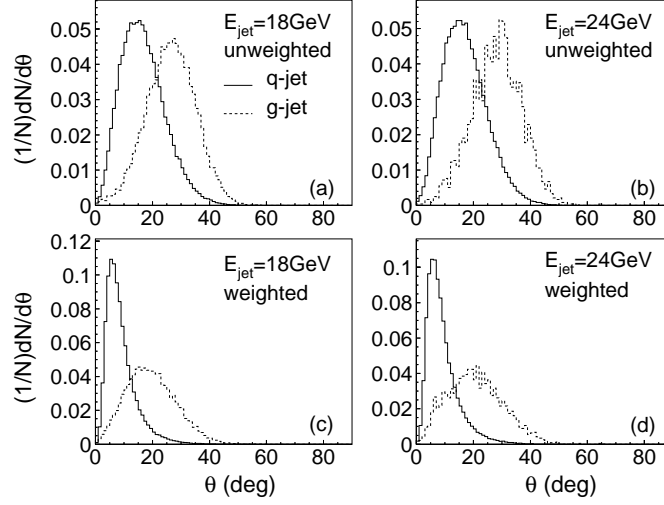


Figure 3: The cone angular distributions of quark jets and gluon jets with energy at (a)(c) 18GeV and (b)(d) 24GeV. (a)(b) without momentum weight, (c)(d) with momentum weight.

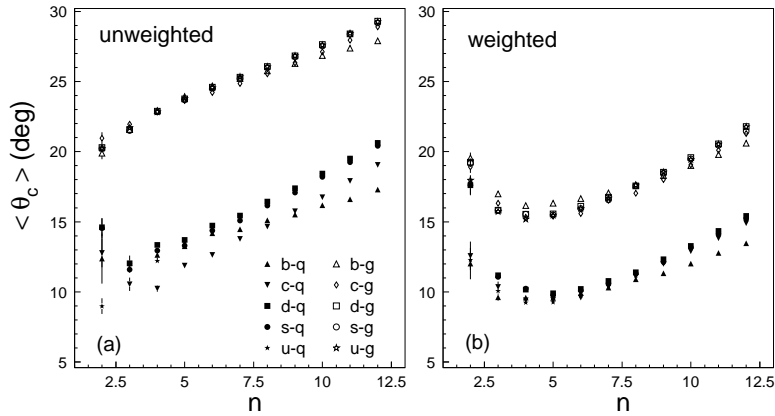


Figure 4: The distributions of average cone angles with different flavors quark jets and gluon jets as functions of charged multiplicities. (a) without momentum weight, (b) with momentum weight

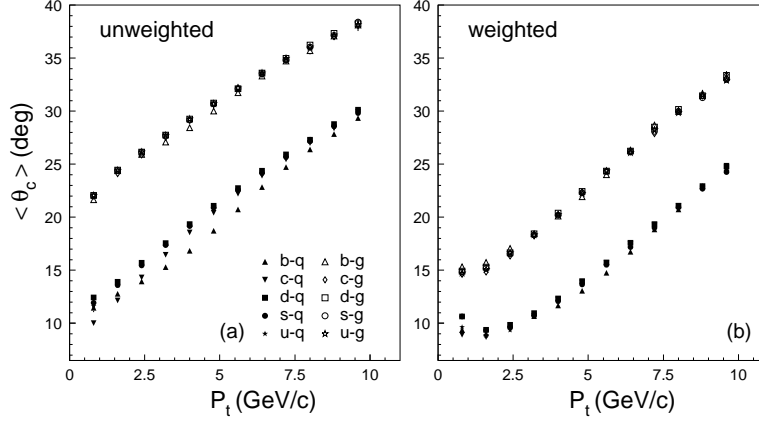


Figure 5: The distributions of average cone angles of quark jets and gluon jets with different flavors as functions of transverse momentum. (a) without momentum weight, (b) with momentum weight.

transverse momentum bin:

$$\langle \theta_C \rangle_{l,k} = \frac{1}{n_l} \sum_{m=1}^{n_l} \langle \theta_C \rangle_{jet-k}^{l,m}, \quad (k = 1, 2, 3; l = 1, 2, \dots, 12). \quad (7)$$

Where n_l is the number of jet in the l th transverse momentum bin, $\langle \theta_C \rangle_{jet-k}^{l,m}$ is the cone angle of the m -th jet of kind k in the l th transverse momentum bin. The results of average cone angle of the gluon jet and quark jet with different flavors *vs.* transverse momentum in each transverse momentum bin for Jetset 7.4 Monte Carlo generators are shown in Fig's 5 (a) and (b), respectively.

It can be seen from the Fig 5 (a) that the average cone-angles of the gluon jet and quark jet *vs.* transverse momentum are differently, and they increases quickly with the increaes in transverse momentum for all flavors, showing a strong positive linearly correlation. The average cone angle of the gluon jet is bigger than that of the quark jet, providing also evidence that the gluon jet is "fatter" than quark jet.

The distributions of cone angles with momentum weight and unweight *vs.* transverse momentum are qualitative similar comparing Fig 4 (a) and (b). When we added momentum weight as Fig 4 (b), their distributions present properly linear positive correlation only in higher transverse momentum area; while in lower area their curves decreased for first one or two points, which may probably be roused by the leading particle effect as same as section 2.

We also see that the distributions of average cone angle of the gluon jet with different flavors *vs.* transverse momentum are superposition within range error, which shows that gluons jet produced by hard gluon emitted by different flavor quarks have certain same characters; while that of the quark jets with different flavors are qualitatively conformable and are quantificationally difference. This kind of difference is mainly aroused by mass

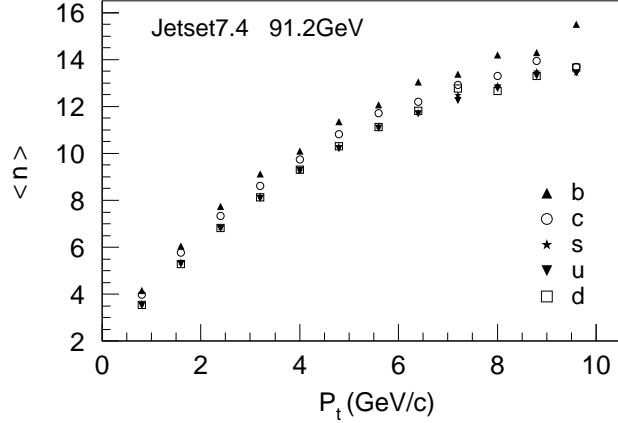


Figure 6: The distributions of multiplicities with different flavors quark jets as functions of transverse momentum.

difference among different flavors quarks.

In order to explain the character difference of cone angular distributions with different flavors quark jets in fig.4 and fig.5, we still use Monte Carlo simulation Jetset 7.4 to produce data, divide the transverse momentum range $P_t = 0.4 \sim 10 \text{ GeV/c}$ into 12 equal bin, and then get distributions of charged multiplicity with different flavors quark jets as functions of transverse momentum, as shown in Fig 6.

From Fig 6, it can be seen that average charged multiplicities of quark jets with different flavors are different in the same transverse momentum bin, distributions of average charged multiplicities arrange to mass magnitude of the original quarks, and the larger the mass the larger the charged multiplicity is. Suppose mass of two kinds of quarks are m_A and m_B , and the final state charged hadron multiplicities are n_A and n_B , respectively. Then we have

$$n_A > n_B, \quad (as \quad m_A > m_B). \quad (8)$$

We introduce a simple model base above result. First we suppose there are only two kinds of particles in jets: one kind is large vertical momentum and small transverse momentum, noted as Γ , and the transverse momentum of each particle is P , the angle between the particle and the jet axis is α ; the other is small vertical momentum and large transverse momentum, noted as Λ , and the transverse momentum of each particle is Q , the angle between the particle and the jet axis is β . Obviously,

$$\alpha < \beta, \quad P < Q. \quad (9)$$

Arbitrarily choose two different kinds of quarks A and B as the research objects, their mass separately are m_A and m_B , and $m_A > m_B$. Jet formed by quark- A is noted as A -jet and jet formed by quark- B is noted as B -jet. Under the same condition of transverse momentum

P_t , suppose the number of Γ particle is n , number of Λ particle is m in A -jet, and suppose the number of Γ particle is N and number of Λ particle is M in B -jet. Then we have

$$P_t = nP + mQ = NP + MQ. \quad (10)$$

The average cone angles of A -jet and B -jet separately are

$$\langle\theta\rangle_{A\text{-jet}} = \frac{n\alpha + m\beta}{n + m}, \quad \langle\theta\rangle_{B\text{-jet}} = \frac{N\alpha + M\beta}{N + M}. \quad (11)$$

Substituting equation (11) with equations (8)-(10), we obtain

$$\langle\theta\rangle_{A\text{-jet}} - \langle\theta\rangle_{B\text{-jet}} = (\alpha - \beta)(nM - Nm) = (\alpha - \beta)\left(\frac{P(n - N)}{Q}\right) < 0. \quad (12)$$

Therefor, we get conclusion that the average cone angle of A -jet is smaller than that of B -jet with the same transverse momentum, i.e, the larger the quark mass the smaller the average cone angle is.

Suppose rates of number of Γ particles and number of Λ particle in one certain kind of jet to the total charged multiplicity of this jet is a constant. And note rate of number of Λ particles in A -jet to charged multiplicity of this jet is x , rate of number of Λ particles in B -jet to charged multiplicity of B -jet is y , thus

$$x = \frac{n}{n + m}, \quad y = \frac{N}{N + M}. \quad (13)$$

From equation (8)-(10), we have $n > N$ and $m < M$. After substituting them into equation (13) we get $x > y$, i.e, proportion of number of particles with large vertical momentum in A -jet is higher than that in B -jet. When charged multiplicities N_{tot} for different flavors quark jets are the same, the average cone angle of jets can be expressed as

$$\langle\theta\rangle_{A\text{-jet}}^{N_{\text{tot}}} = N_{\text{tot}}[x\alpha + (1 - x)\beta], \quad (14)$$

$$\langle\theta\rangle_{B\text{-jet}}^{N_{\text{tot}}} = N_{\text{tot}}[y\alpha + (1 - y)\beta]. \quad (15)$$

Thus slopes of distribution curves of average cone angles as functions of charged multiplicities can be got according to the following equations

$$\frac{\partial\langle\theta\rangle_{A\text{-jet}}^{N_{\text{tot}}}}{\partial N_{\text{tot}}} = x\alpha + (1 - x)\beta, \quad (16)$$

$$\frac{\partial\langle\theta\rangle_{B\text{-jet}}^{N_{\text{tot}}}}{\partial N_{\text{tot}}} = y\alpha + (1 - y)\beta. \quad (17)$$

For $x > y$, thus

$$\frac{\partial\langle\theta\rangle_{A\text{-jet}}^{N_{\text{tot}}}}{\partial N_{\text{tot}}} < \frac{\partial\langle\theta\rangle_{B\text{-jet}}^{N_{\text{tot}}}}{\partial N_{\text{tot}}}.$$

Namely slope of distribution for A -jet is smaller than that for B -jet. For $m_A > m_B$, this means that the larger the mass of quark for producing jet the smaller the distribution slope of the average cone angle as function of jet transverse momentum.

For five different flavors quarks b, c, s, u, d , mass of them exist difference, i.e, the mass of quark b (5GeV) is much larger than mass of other flavors quarks, next is quark c (1.5GeV), quark u, d has the least mass. In summery: (1) The distributions of average cone angles *vs.* charged multiplicities with different flavors quark jets are different, shown in fig 4 (a) and (b). For mass of quark- b is the largest, slope of its distribution is the least. (2) The distributions of average cone angles *vs.* transverse momentum among different flavors quark jets is some difference, shown in fig 5 (a) and (b). Since mass of quark- b is the largest, the average cone angle distributions is the lowest.

6 Conclusion

In this paper, we produced the data using Monte Carlo simulation Jetset7.4 in e^+e^- collision events with the center of mass energy at 91.2GeV, selected out 3-jet events with Durham algorithm and identified quark jets and gluon jets using angular method. We defined angular variables α_1, α_2 for qualitatively describing the angular distribution characters inside jets and defined cone angles of jets for quantificationally describing the geometrical characters of particles inside jets.

The cone angle distribution of gluon jet at the same energy is obviously wider than that in quark jets. The average cone angles of gluon jets, at the same multiplicity or the same transverse momentum, are much larger than that of quark jets. This is in agreement with the predictions of QCD theory that gluons are "fatter" than quarks, providing further evidence that the gluon jet is "fatter" than quark jet.

The average cone angles of gluon and quark jet without weight increases with charged multiplicities or transverse momentum, i.e, they present linear positive correlations. This illustrates that cone angle can reflect the distribution characters of particles inside jets, and can also reflect transverse momentum distribution characters, namely, we can use this geometrical characters to describe the dynamical characters inside jets.

The distributions of average cone angle of gluon and quark jet with momentum weight as functions of charged multiplicity or transverse momentum all have a minimum value. Namely, with the increasing of charged multiplicity or transverse momentum, the average cone angle decreases at first, then increases when $n > 4$ or $p_t > 2\text{GeV}$. The appearance of minimum value is aroused by the leading particle effect.

The distribution of average cone angles of the gluon jets with different flavors *vs.* multiplicity or transverse momentum is same, while that of the quark jets has distinctions, which is as a result of that the mass difference of different flavors quarks raised the symmetrical broken of strong interaction.

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